3

90636

906360

NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Calculus, 2008

90636 Integrate functions and use integrals to solve problems

Credits: Six
9.30 am Tuesday 18 November 2008

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show the results of any integration needed to solve the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**	
Integrate functions and use integrals to solve problems.	Use advanced integration techniques to find integrals and solve problems.	Solve more complex integration problem(s).	
Overall Level of Performance			

You are advised to spend 50 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

Find the integrals.
You do not need to simplify your answers.
Do not forget the arbitrary constant.

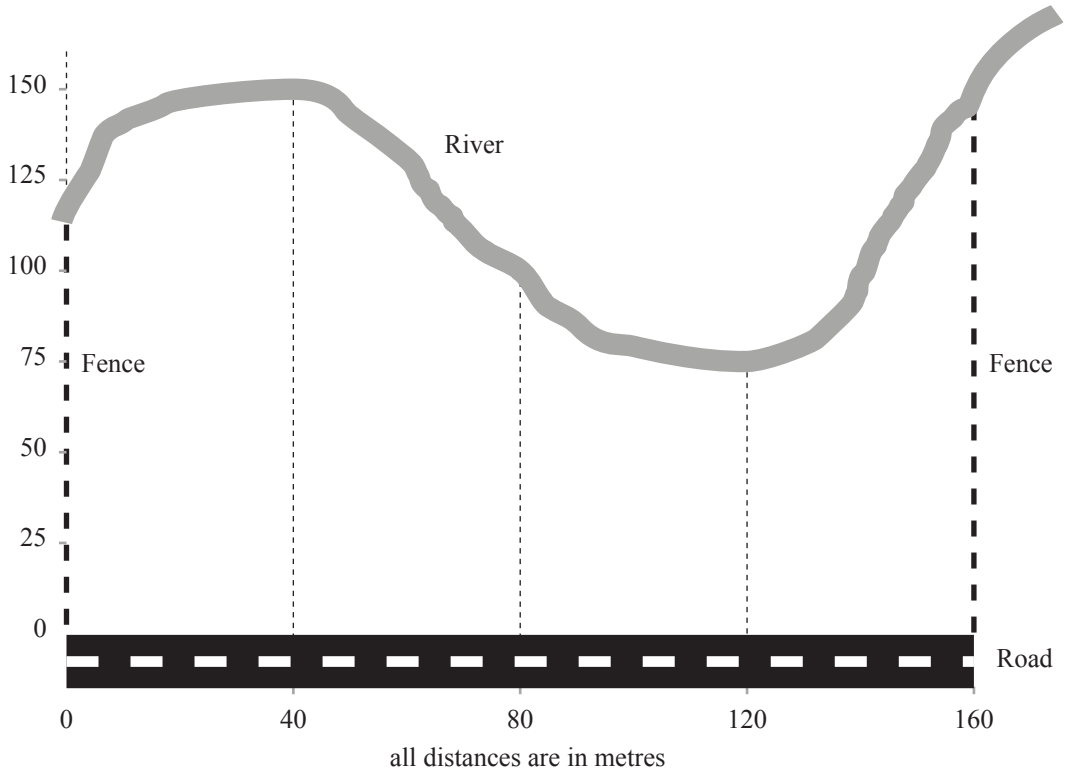
(a) $\int (2-3x)^5 \mathrm{d}x$

(b) $\int \left(\frac{3}{x^3} - \frac{4}{x} \right) \mathrm{d}x$

QUESTION TWO

Assessor's use only

The diagram shows a paddock bounded by a river, a road and two fences perpendicular to the road. A farmer wants to know the area of the paddock. He measures the perpendicular distance from the road to the river every 40 m.



The measurements are in the table below:

distance along the road (m)	0	40	80	120	160
perpendicular distance to river (m)	110	150	100	75	125

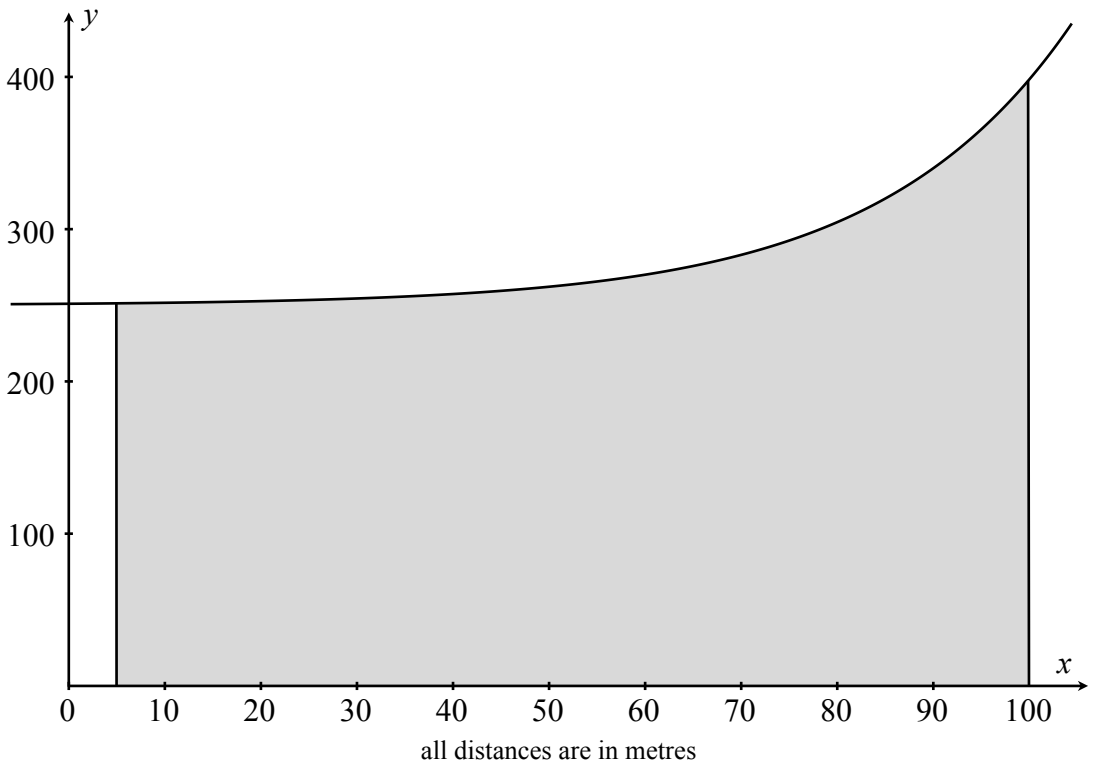
Use either the Trapezium rule or Simpson's rule to estimate the area of the paddock, using 4 sub-intervals.

QUESTION THREE

Assessor's use only

Another paddock can be modelled by the shaded area on the graph of $y = e^{0.05x} + 250$ where x and y are measured in metres.
The shaded area is bounded by $y = 0$, $y = e^{0.05x} + 250$, $x = 5$ and $x = 100$

Use integration to calculate the area of the paddock.



all distances are in metres

Give the results of any integration needed to solve this problem.

QUESTION FOUR

Assessor's use only

A tank initially contains 10 litres of water.

It is filling at the rate of $\dfrac{50}{(t+1)^2}$ litres/minute where t is the time in minutes.

How much will it contain after one hour?

Give the results of any integration needed to solve this problem.

QUESTION FIVE

Assessor's use only

Find the integral: $\int \frac{3x+8}{\sqrt{x-1}}\,dx$

A suitable substitution may be helpful.

QUESTION SIX

Assessor's use only

An object's acceleration is given by the formula:

$$a(t) = 0.6e^{0.2t}$$

where a is the acceleration of the object (m s^{-2})
and t is the time since the start of the object's motion (seconds).

If the object had a velocity of 5 m s^{-1} after 2 seconds, how far did it travel during its fourth second of motion?

Give the results of any integration needed to solve this problem.

QUESTION SEVEN

Assessor's use only

The rate at which a drug is used up by the human body is proportional to the concentration of the drug in the bloodstream.

Carbimazol is a drug given to treat patients with a hyperactive thyroid.

Carbimazol has a half-life of 8 hours – this means that it takes 8 hours for a patient's body to use up half the Carbimazol in their bloodstream.

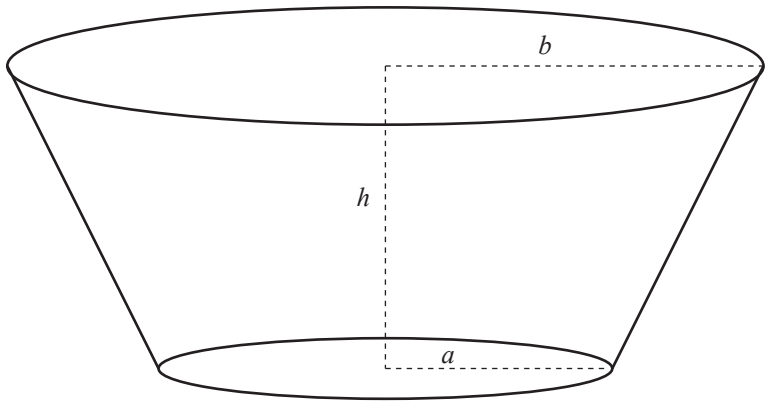
Soon after the usual dose is taken, the concentration of Carbimazol in the patient's blood is 16 mg per litre. When the concentration drops below 5 mg per litre, the drug is no longer effective.

Write a differential equation, and solve it to find out how long the drug is effective.

QUESTION EIGHT

Assessor's use only

A symmetrical plant pot has a circular base, radius a cm, a circular top, radius b cm, and straight sloping sides. Use integration to find the volume of the pot in terms of a, b and h.



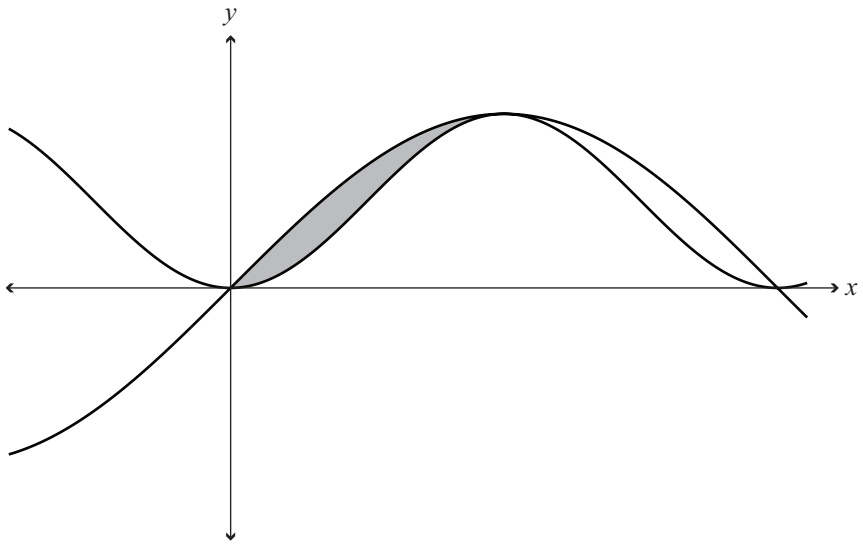
QUESTION NINE

Assessor's use only

The shaded region in the graph below is bounded by the two functions:

$y = \sin kx$
$y = \sin^2 kx$

where k is a natural number



Find the area of the shaded region in terms of k.

Give the results of any integration needed to solve this problem.

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number